Gait planning for biped locomotion on slippery terrain

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Abstract— **We propose a new biped locomotion planning method that optimizes locomotion speed subject to friction constraints. For this purpose we use approximate models of required coefficient of friction (RCOF) as a function of gait. The methodology is inspired by findings in human gait analysis, where subjects have been shown to adapt spatial and temporal variables of gait in order to reduce RCOF in slippery environments. Here we solve the friction problem similarly, by planning on gait parameter space: namely foot step placement, step swing time, double support time and height of the center of mass (COM). We first used simulations of a 48 degrees-of-freedom robot to estimate a model of how RCOF varies with these gait parameters. Then we developed a locomotion planning algorithm that minimizes the time the robot takes to reach a goal while keeping acceptable RCOF levels.**

Our physics simulation results show that RCOF-aware planning can drastically reduce slippage amount while still maximizing efficiency in terms of locomotion speed. Also, according to our experiments human-like stretched-knees walking can reduce slippage amount more than bent-knees (i.e. crouch) walking for the same speed.

I. INTRODUCTION

While walking, humans avoid slipping by two levels of locomotion control: a high-level, anticipatory control cycle which adjusts gait variables (e.g. walking at slower pace, with shorter steps, stiff limbs) [1], [2], [3], and a low-level, reactive control of the limbs (e.g. locally adapting arm swing for short-term stability) [3]. Very promising advances in friction-constrained biped robot locomotion have focused on the second approach, providing low-level controllers for a given robot gait [4], [5], [6]. While reactive control can help reduce tangential forces locally, it may not be sufficient in very low friction surfaces. For example a robot with rubber soles would be subjected to around 0.15 kinetic friction when walking on ice, or even less if the soles themselves were covered in ice. Slipping can be reduced in such low friction floors without changing gait, but not eliminated [4].

In this paper we complement previous studies on slippery terrain locomotion control from the other perspective: anticipatory control (i.e. planning) of gait. Compared to footstep planning research [7], [8], [9] we do not assume fixed step timing, but plan both foot placement, timing (i.e. step swing time, double support time) and reference COM height. These variables have been shown to be controlled by humans when walking on slippery surfaces [1], [2], [3] in order to lower the Required Coefficient of Friction (RCOF) for a slip to occur [2]. Inspired by those findings, our idea is to estimate a model of RCOF as a function of gait, for each walking control policy used by the robot. This model can then be used to plan gait trajectories that keep expected tangential and normal forces within acceptable limits. For this purpose we assume that COF estimates of the walking surface are available, computed for example from visual sensors.

Our contributions are: 1) We estimated models of RCOF as a function of gait parameters for a human-size, 48 degrees of freedom (DOF), biped robot and two types of gait (stretched-knees and bent-knees gait). We parameterized gait with 4 variables: step displacement, step swing time, double-support time and COM height. Contour plots of RCOF as a function of these variables are shown in Section IV. 2) We propose a new gait planning algorithm that minimizes total walking time subject to RCOF constraints. Experimental results were obtained in simulation (Section V), with a setup similar to the one used to evaluate previous related controllers [4]. The results show how gait planning with RCOF models can drastically reduce slippage amount, and how it can be further reduced with stretched-knees than with bent-knees gait.

II. BACKGROUND

Research in gait, physiology and ergonomics has shown that humans adopt a specific, cautious gait when there is awareness of a slippery surface [1], [2], [3]. On the other hand, when an estimate of the coefficient of friction also exists from visual input and/or experience, both gait and muscle activation patterns become characteristically different since the first step on the surface. This indicates an anticipation strategy and not a reactive adaptation of normal gait [3]. In this paper we interchangeably refer to this process as anticipatory control of gait, or planning of gait. When there is anticipation, humans increase swing and double-support times, take shorter steps, reduce foot velocity at contact and increase limb stiffness [1], [2], [3]. All these variables have been shown to be used to regulate the Required Coefficient of Friction (RCOF): the ratio of shear to normal ground reaction force [2]. The anticipatory control of gait is complemented by a reactive control of the limbs to better maintain stability, usually manifested by high arm oscillations in the sagittal and frontal planes [3] and very reactive leg movements.

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Biped robot control has focused on the reactive, low-level, control approach to reduce slipping and maintain stability of robots. For instance, friction cones have been used as constraints in the optimization approach to inverse dynamics [10] or in the operational space control framework [6]. Such approaches basically change inter-limb coordination reactively so that friction and stability constraints are met, while keeping gait fixed or as close as possible to the normal mode. On the other hand, design parameters in the preview controller [11] can be slightly tuned to reduce the RCOF for a fixed gait, and feedback ZMP controllers manually adapted to account for friction [4]. Efforts have also been put into reactive reflex controllers that, without changing gait parameters, try to reduce slipping after it is detected (e.g. by waist or foot acceleration reflexes [5]).

In our work we tackle the slippery terrain problem with gait planning. Our purpose is to eliminate slipping as much as possible by changes in gait. Such approach solves the known problem of reactive controllers to not be able to avoid slipping on fast gait [5], by changing gait speed itself and other gait variables. The approach is similar to "footstep planning" algorithms, where a sequence of footsteps is computed given a 3D map of the world using graph search [7], [8] or others [9]. Gait planning is an extension to "footstep planning" in the sense that both foot placement and other gait variables are planned as well.

III. METHODOLOGY

We define gait planner as a function that computes a sequence of footsteps with associated gait parameters, given initial and final robot configurations and a description of the environment. We assume that an estimate of ground friction is either known or used as a design variable in the planner. In this paper we consider only bipedal gait. The sequence of steps is represented by a matrix $\mathbf{S} \in \mathbb{R}^{N \times G}$, where N is the number of steps and G the dimension of a step vector. In this paper, the ith step vector $\mathbf{s}^{(i)} \in \mathbb{R}^G$ is defined as

$$\mathbf{s}^{(i)} = (\mathbf{x}^{(i)}_{feet}, \Delta\mathbf{x}^{(i)}_{feet}, \boldsymbol{\alpha}^{(i)}_{feet}, \Delta\boldsymbol{\alpha}^{(i)}_{feet}, \Delta t^{(i)}_{sw}, \Delta t^{(i)}_{ds}, z^{(i)}_{COM}).$$

$\mathbf{x}^{(i)}_{feet}$ and $\Delta\mathbf{x}^{(i)}_{feet}$ represent the initial position of the feet and the position change during that step, respectively, while $\boldsymbol{\alpha}^{(i)}_{feet}$, $\Delta\boldsymbol{\alpha}^{(i)}_{feet}$ represent orientation and orientation change. Only one foot is displaced at each $\mathbf{s}^{(i)}$ (i.e. half of the entries in $\Delta\mathbf{x}^{(i)}_{feet}$ and $\Delta\boldsymbol{\alpha}^{(i)}_{feet}$ are zeros). $\Delta t^{(i)}_{sw}$ is the swing time (i.e. time spent with the foot in the air), $\Delta t^{(i)}_{ds}$ the double support time (i.e. time spent with both feet on the ground), and finally $z^{(i)}_{COM}$ the desired height of the COM. A low-level controller is then responsible for generating and tracking a trajectory for all joints which guarantees dynamic stability and the gait task $\mathbf{S}$.

Let us assume that the low-level controller leads to a total force applied on the contact surface $\mathbf{F}(t)$. The maximum coefficient of friction between the feet and the contact surface that leads to a slip during a time interval I is given by

$$RCOF(I) = \max_{t \in I} \left|\left| \frac{F_T(t)}{F_N(t)} \right|\right|, \quad (1)$$

and is called "required coefficient of friction" (RCOF). Here F_T and F_N refer to the tangential and normal components of the force, respectively.

A. RCOF model

Our proposal is to estimate an approximate model of RCOF for a single step, as a function of the step's parameters: $\widehat{RCOF}(\mathbf{s}^{(i)})$. If we consider a step $\mathbf{s}^{(i)}$ occurring at time $t^{(i)}$, its RCOF can be obtained by using equation (1) on the interval $I = [t^{(i)}; t^{(i)} + \Delta t^{(i)}_{sw} + \Delta t^{(i)}_{ds}]$. Assuming perfect tracking of force and joint references by the low-level controller, the force $\mathbf{F}(t)$ (and hence RCOF) still depends not only on the motion of the robot during I but also on the robot's state at $t^{(i)}$ and $t^{(i+1)}$, to accommodate previous and future motion. Estimating $RCOF(I)$ is thus as complex as defining the low-level controller itself. To simplify the problem, we make the approximation that the state of the robot at the beginning and at the end of a step are entirely defined by $\mathbf{s}^{(i)}$ (which includes initial and final feet state). While this assumption may only be valid in cyclic walking or static walking, it makes the problem tractable by allowing to define a RCOF prediction as a function of gait parameters only: $\widehat{RCOF}(\mathbf{s}^{(i)})$. We discuss possible extensions in Section VI.

At the moment, our implementation of the RCOF model consists of a discretized lookup table $\widehat{RCOF} : \mathbb{R}^G \rightarrow [0; \infty)$ for fast access. The function is learned by sampling the input space and computing RCOF values as in equation (1), from the force references given by the low-level controller. Details on the controller are given in Section IV.

B. Gait planning

We use the estimated RCOF model for gait planning by a constrained optimization problem. Specifically, we minimize the total walking time with a constraint on RCOF. Note that simply minimizing the RCOF would lead to minimizing walking speed (i.e. the best way not to slip is not to move), which is not desirable. Assuming the contact surface's static coefficient of friction (COF) μ is known, we solve

$$\begin{aligned} \underset{\mathbf{S}}{\text{minimize}} \quad & \sum_{i=1}^{N} \Delta t^{(i)}_{sw} + \Delta t^{(i)}_{ds} \\ \text{subject to} \quad & \widehat{RCOF}(\mathbf{s}^{(1)}) < \mu^{(1)} \\ & \vdots \\ & \widehat{RCOF}(\mathbf{s}^{(N)}) < \mu^{(N)} \end{aligned} \quad (2)$$

to obtain a final gait plan $\mathbf{S}$.

In practice, the problem can be split into optimization of foot placement followed by the rest of the variables. This way, currently popular footstep planning approaches through A* graph search [7] can still be used. In that case $(\mathbf{x}^{(i)}_{feet}, \Delta\mathbf{x}^{(i)}_{feet}, \boldsymbol{\alpha}^{(i)}_{feet}, \Delta\boldsymbol{\alpha}^{(i)}_{feet})$, $i = 1...N$, are obtained by a footstep planner given the world geometry constraints (e.g. obstacles, holes, roughness), and a similar but simpler problem to (2) can be solved. That new problem has $(\mathbf{x}^{(i)}_{feet}, \Delta\mathbf{x}^{(i)}_{feet}, \boldsymbol{\alpha}^{(i)}_{feet}, \Delta\boldsymbol{\alpha}^{(i)}_{feet})$ for $i = 1...N$ as additional

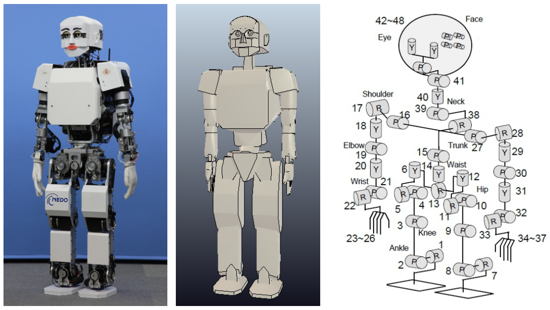

Fig. 1. The RCOF models and experimental results were obtained on a 48 DOF humanoid robot model: KOBIAN [12]. For physics simulations we used the Open Dynamics Engine on the V-REP simulator (middle image).

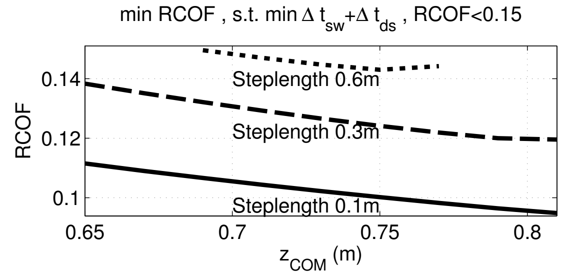

Fig. 2. Results on the bent-knees controller. When all other gait variables are fixed, a higher COM can further optimize RCOF.

constraints, while the optimization variables are the remaining gait parameters $(\Delta t_{sw}^{(i)}, \Delta t_{ds}^{(i)}, z_{COM}^{(i)})$. This corresponds to solving N simple optimization problems:

$$\begin{aligned} \underset{\mathbf{s}^{(i)}}{\text{minimize}} \quad & \Delta t_{sw}^{(i)} + \Delta t_{ds}^{(i)} \\ \text{subject to} \quad & \widehat{RCOF}(\mathbf{s}^{(i)}) < \mu^{(i)}. \\ & \mathbf{x}_{feet}^{(i)}, \Delta \mathbf{x}_{feet}^{(i)}, \boldsymbol{\alpha}_{feet}^{(i)}, \Delta \boldsymbol{\alpha}_{feet}^{(i)} \end{aligned} \tag{3}$$

Finally, we consider the hypothesis that several solutions exist for the previous problem. For example, notice that changing the height of the COM may not force a reference walking speed to change. This parameter thus gives redundancy to the optimization problem, and it can be chosen such as to further minimize the RCOF. The final problem, which we used for the results in this paper, then becomes the following bilevel optimization problem:

$$\begin{aligned} \underset{\mathbf{s'}^{(i)}}{\text{minimize}} \quad & \widehat{RCOF}(\mathbf{s'}^{(i)}) \\ \text{subject to} \quad & \\ & \mathbf{s'}^{(i)} \in \underset{\mathbf{s}^{(i)}}{\text{argmin}}(\Delta t_{sw}^{(i)} + \Delta t_{ds}^{(i)}). \\ & \widehat{RCOF}(\mathbf{s}^{(i)}) < \mu^{(i)} \\ & \mathbf{x}_{feet}^{(i)}, \Delta \mathbf{x}_{feet}^{(i)} \\ & \boldsymbol{\alpha}_{feet}^{(i)}, \Delta \boldsymbol{\alpha}_{feet}^{(i)} \end{aligned} \tag{4}$$

IV. THE RCOF MODEL OF A FULL-SIZE HUMANOID ROBOT

We estimated the RCOF model of a full-size humanoid robot with 48 degrees of freedom (DOF). In particular, we used the robot model of KOBIAN [12], which is shown in Figure 1. Its structure is similar to that of other full-size humanoids such as HRP-2, with the exception of a more human-like pelvis joint and head DOF.

We obtained RCOF models using two different low-level controllers. One achieves stretched-knees walking and the other bent-knees (i.e. crouch) walking. In both cases, dynamic stability is guaranteed by a FFT-based pattern generator [13] which, similarly to the preview controller [11], adjusts COM trajectory so that a reference ZMP trajectory is met. ZMP references were set as to lie on the center of the stance foot during the swing phase, and to shift to the other foot during the double support phase using spline interpolation. Joints are position controlled. We remind that our approach is still valid for other low-level controller options, such as recent optimization-based inverse dynamics approaches for torque-controlled robots [10] or others. The total force applied by the robot on the ground $\mathbf{F}(t)$ was computed from the equations of motion of all links of the robot, which are in turn given by the low-level controller. The RCOF was computed from $\mathbf{F}(t)$ as in equation (1). In this paper we show results only for forward walking, although the methodology is not limited to this option in any way.

To estimate $\widehat{RCOF}(s^{(i)})$, we sampled its input space (i.e. gait parameters) uniformly such that:

- Step length $||\Delta \mathbf{x}_{feet}^{(i)}|| \in [0.10; 0.60]$ meters, samples every 0.10m
- Step swing time $\Delta t_{sw}^{(i)} \in [0.81; 2.70]$ seconds, samples every 0.09s
- Double support time $\Delta t_{ds}^{(i)} \in [0.09; 0.90]$ seconds, samples every 0.09s.

The parameter $z_{COM}^{(i)}$ is controller-dependent and so we discuss it in the following respective sections.

A. RCOF model with bent-knees walking

The bent-knees controller computes joint commands using inverse kinematics and taking COM height as an input [13]. We directly use the parameter $z_{COM}^{(i)}$ of a given step $\mathbf{s}^{(i)}$ as an input to the controller. We sampled the parameter every 0.02 meters inside the interval $z_{COM}^{(i)} \in [0.65; 0.85]$.

We show the estimated model in Figure 2 and 3. Figure 2 exemplifies the behavior of RCOF in the upper level problem of equation (4). After walking speed is maximized subject to friction constraints ($\mu = 0.15$ in Figure 2) and given step placement as input, the variable $z_{COM}^{(i)}$ is redundant and can be used to further minimize RCOF (e.g. from 0.15, to around 0.143 when step length is 0.60 meters). In general we found that the higher the COM, the lower the RCOF (and thus lower chances of slipping).

Figure 3 shows contour plots of the RCOF model as a function of the different gait parameters, for bent-knees walking. On the left and middle we show how RCOF varies with step timing variables, for fixed step length. The RCOF at each point depends also on COM height, and so the minimum RCOF across different heights is shown at each point. RCOF

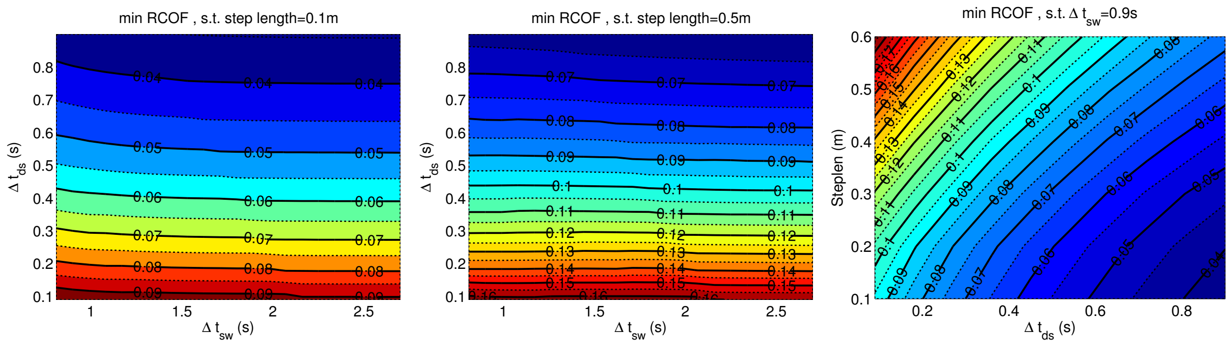

Fig. 3. The RCOF model using bent-knees walking. Left and middle: step length is fixed; Right: step swing time is fixed.

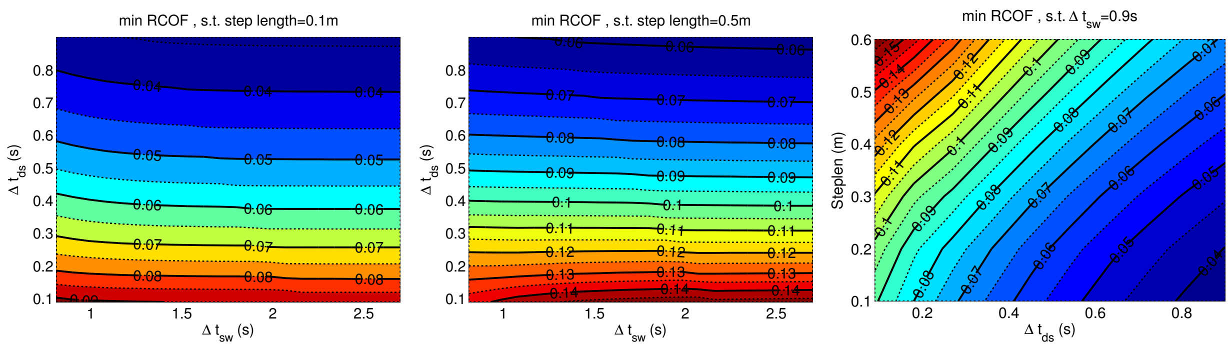

Fig. 4. The RCOF model using stretched-knees walking. Left and middle: step length is fixed; Right: step swing time is fixed.

is mainly dependent on the time spent on double support as contour lines are almost horizontal in the figure. This is explained by the fact that large forces have to be used to bring the center of pressure from one foot to the other during the double support phase of walking. Also notice how walking on very low-friction terrain without slipping requires a very long double support (e.g. around 0.65 seconds for $\mu = 0.08$, on a long half-a-meter step). On the right of the same figure, we show how RCOF varies with step length and double support time, for fixed step swing time. Not only is there an optimum double support time for each step length choice, but this relationship is also not straightforward. Also note that previous literature regarding walking on slippery terrain use gait parameters that lead to very dangerous RCOF. For example in Kajita's pioneer low-level controller work [4], step length was set at 0.60 meters and very short double support phases were used (0.1s), while walking on a floor with $\mu = 0.08$. A special feedback controller is used in that publication to adapt the COM trajectory while keeping fixed gait parameters, even though slipping is not suppressed. We remind that our argument in this paper is that such low-level approaches can be complemented by adapting gait as well and thus avoid slipping where that requirement applies. We will go back to this idea in another experiment in Section V.

B. RCOF model with stretched-knees walking

Stretched-knees walking becomes required when a more human-like walking style is desired for a humanoid robot. In this case the robot's knees should completely stretch when standing still or when its feet impact the ground. While knee angle trajectories indirectly define COM height, in stretched-knees walking this height is not constant along a step (contrary to bent-knees walking) but varies much like an inverted pendulum. Maximum height is achieved around middle swing, while step-length and the bending knee angle affect COM height excursion.

Our stretched-knees controller uses inverse kinematics where knee angle trajectories are given as input. The controller was discussed in [14]. We define the knee angle ϕ as the angle between the upper and lower leg, such that $\phi = 0$ degrees for fully stretched knees and for example $\phi = 90$ degrees for a right angle between upper and lower leg. In this controller we decided to indirectly define $z^{(i)}_{COM}$ by controlling two variables $\phi^{(i)}_{stancebend}$ and $\phi^{(i)}_{swingbend}$: the maximum angles of contraction of the stance leg's knee and of the swing leg's knee, respectively. Full trajectories of the knees were obtained by spline interpolation between stretch (1 degree) and bend, which happen at impact and just after double support, respectively. To estimate $\widehat{RCOF}(\mathbf{s}^{(i)})$, we sampled the knee angles every 10 degrees in the interval $[5; 45]$. We empirically found this discretization to provide a good enough approximation of the model. The RCOF model for stretched-knees walking is then a function of step-length, timing variables and two knee flexion variables.

Figure 4 shows the RCOF model for stretched-knees walking. The contour plots are similar to bent-knees walking (Figure 3), except for small differences. The main structure is maintained: mostly horizontal lines on the swing-double support plot, and quarter circumferences in the double support-step length plot. The most notable difference is that there is a vertical shift in both plots, with respect to bent-knees walking. For the same gait parameters, a lower RCOF can be achieved when walking in a human-like, knee stretching, pattern than when knees are bent and COM height is constant along a step. The improvement in RCOF is approximately 0.02. We believe this improvement comes from the fact that in bent-knees walking the COM height is restricted, or constant height cannot be accomplished: especially for long steps. On the other hand, if the knees are stretched for a pendulum-like motion of the COM, both height and step length can be more flexibly exploited to improve RCOF. As in bent-knees walking (Section IV-A), we also found that a higher COM allows for lower RCOF.

V. EXPERIMENTAL RESULTS: GAIT PLANNING USING RCOF MODELS

In this section we report on experimental results of the proposed gait planning method which uses gait-dependent RCOF models (Section III-B). The approach was validated using physics simulation software (Open Dynamics Engine, V-REP Simulator), again using the robot model of KOBIAN.

A. The advantage of gait planning

To stress the advantage of planning gait parameters other than footstep placement, we compare walking results obtained with our gait-planner to results obtained with normal walking gait. For "normal gait" we used the same parameters as in [4], except for $z^{(i)}_{COM}$ which had to be lowered to make the motion feasible on our robot model: $||\Delta\mathbf{x}^{(i)}_{feet}|| = 0.60$m; $\Delta t^{(i)}_{sw} = 0.70$s; $\Delta t^{(i)}_{ds} = 0.10$s; and $z^{(i)}_{COM} = 0.73$m (bent-knees controller). Such normal walking gaits are quite fast for low-COF terrains such as the ones used in [4] ($\mu = 0.08$). This fact makes it difficult to completely avoid slipping. For instance in our robot model, of similar weight to the HRP-2, the RCOF is approximately 0.19, while in [4] the RCOF is between 0.13 and 0.19 depending on a design parameter. These RCOF values are higher than $\mu = 0.08$ and justify the high amount of slipping reported in that paper. Even though low-level feedback controllers can be used to reduce slipping or keep stability, here we attempt to avoid slipping itself as much as possible through gait.

Figure 5 (top) shows the feet trajectory while contacting the ground, as well as the waist, obtained with normal walking gait and $\mu = 0.08$. High amount of slipping is visible. On the other hand, using our gait planner (4) with the same footstep positions leads to the gait parameters $\Delta t^{(i)}_{sw} = 0.81$s and $\Delta t^{(i)}_{ds} = 0.81$s. While speed is 1.8 times lower, $\widehat{RCOF} < \mu$ holds and as such slippage is minimized. This can be seen clearly in Figure 5 (bottom). Also, our gait planner forces longer double support phases which create a slower ZMP trajectory. Thus the motion becomes more static and the waist follows a sharp but slower trajectory.

Finally, we quantify slipping amount for different μ of the floor-feet in Figure 6. As proposed by [4], we use the SlipIndex measure, $\int |waist_{ref}(t) - waist(t)| dt$. For the

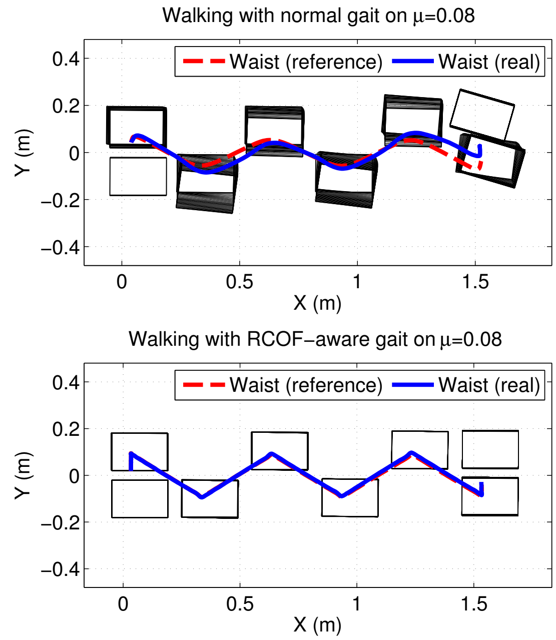

Fig. 5. Results of walking experiments with a normal walking gait (top) and with gait parameters obtained with our planner (bottom).

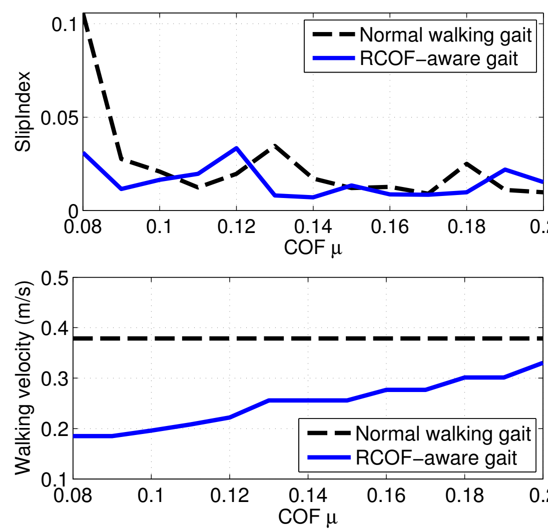

Fig. 6. Experimental results of bent-knees walking for different COF. Slippage amount (top) and reference walking velocity (bottom) are shown for a normal walking gait and for gait parameters obtained with our planner

results in Figure 6, step placement is fixed (step length = 0.60m), while the rest of the gait parameters is planned according to (4), and "normal gait" is fixed as described in the previous paragraph. Noticeably, the SlipIndex is kept considerably lower with our planner when μ is low, and is similar to the "normal gait" for $\mu > 0.10$. Also, notice that as the coefficient of friction gets higher, our planner allows the robot to walk faster as seen in the bottom of Figure 6.

B. The advantage of human-like stretched-knees walking

In another experiment we compared simulation results obtained with our gait-planner when using 1) bent-knees

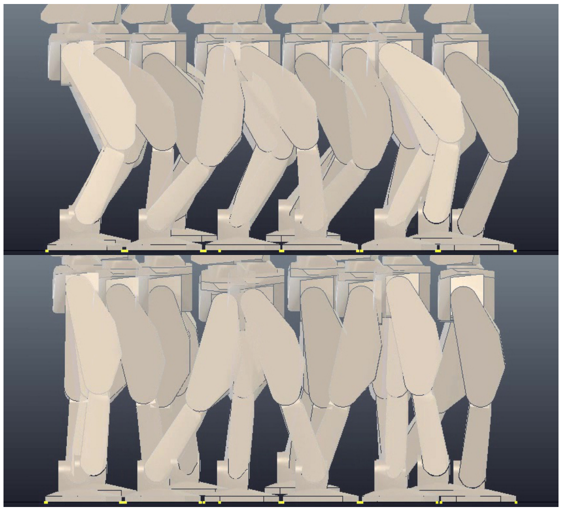

Fig. 7. The bent-knees (top) and stretched-knees (bottom) walking patterns on a $\mu = 0.08$ floor, as seen in the V-REP simulator. SlipIndex was 0.03 for both patterns (negligible, slippage not visible in the figure), but stretched-knees is 6% faster.

walking, and 2) stretched-knees walking controllers. Figure 7 shows the two patterns in simulation, for $\mu = 0.08$, step-length 0.40m and other gait parameters planned according to (4). Slippage is negligible and not visible in the figure. The obtained SlipIndex was equivalent in both (0.03), while stretched-knees walking allowed the robot to walk slightly faster ($\Delta t_{sw} + \Delta t_{ds} = 1.53$s with bent-knees, 1.44s with stretched-knees - around 6% speed increase).

VI. CONCLUSIONS AND DISCUSSION

Inspired by recent human gait literature, we estimated models of required coefficient of friction (RCOF) as a function of gait variables. The models were estimated for a 48 DOF robot using two different controllers (stretched-knees and bent-knees). Based on these models we proposed a gait planning method that can better avoid slipping on low-friction terrain. We validated the methodology in physics simulation software, which showed promising results. We show that our approach can completely avoid slipping while still maximizing speed. This contrasts with research on low-level controllers, which cannot prevent high slippage on very low-friction ground because of the high speed of usual gait patterns. We also found that stretched-knees walking can slightly improve walking results when compared to bent-knees, by either reducing the RCOF or increasing speed for the same RCOF.

While in this paper we considered foot placement, timing and COM height as planning variables, the approach can be extended to other variables. For example, joint stiffness and foot contact velocity are very interesting possible extensions of the approach, since they are anticipatorily controlled by humans [1], [2], [3]. On the other hand, some limitations of the method could be addressed in the future. For example, the current gait-dependent RCOF model ignores the initial state of the robot and considers only one step. A possible extension could be to include an external force in the model $\widehat{RCOF}(\mathbf{s}, \mathbf{F}_{ext})$ for object carrying applications, or even a list of the previous and next step $\widehat{RCOF}(\mathbf{s}^{(i-1)}, \mathbf{s}^{(i)}, \mathbf{s}^{(i+1)}, \mathbf{F}_{ext})$. This would come, however, at the cost of complexity and computation time.

The low-level controllers used for the results in this paper (see Section IV) deal with the redundancy in the system by predefining either COM height or knee angle trajectories. These are directly computed by the planner, which selects them such as to minimize RCOF. In the future we would like to consider more flexible controllers, such as optimization-based inverse dynamics, where redundancy would be solved by weights [10] or hierarchical priorities [6] given to a COM trajectory task and a RCOF-minimization task.

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